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The Entropy of the Quark Gluon Plasma at Finite Temperature

Namani Srinivas[1*], *Sivaraman* Baskar[1], *Aadi* Nagender Babu, *Lavish* Kansal

[1]Department of Physics, KGRCET-Hyderabad- 501504, Telangana, India
[2]Department of CSE, GRIET, Hyderabad, Telangana, India
[3]Lovely Professional University, Phagwara, Punjab, India.

Abstract. In this paper we have calculated the thermal partition function of the thermo- dynamical system of Quark Gluon Plasma and also we have calculated the entropy of quark gluon plasma for the zero chemical potential at the thermal equilibrium condition.

Keywords: QGP, Thermal equilibrium, Entropy, Phase Transition

1 Introduction

There have been many revolutions in the last century in various fields of science especially in physics. One of the most important breakthrough in recent years is the discovery of Quark Gluon Plasma. The word plasma means something moulded. It was used for the first time by Tonks and Langmuir in 1929 on based on its analogy with boold plasma.

The quark gluon plasma is a new phase of hadronic matter at very high density and high temperature. The existence of this was predicted soon after the discovery of the property of asymptotic freedom of QCD (or Non-Abelian) field theory. Later it was realized that QGP is not just a theoretical object, appearing in the early Universe and the collapsing stars, but it can be produced in the laboratory by means of high energy collision of heavy ions. In 1964 Murray Gell-Mann and George Zweig proposed independently that the differences and similarities observed among various hadrons discovered could be easily explained if they were considered as made up of quarks. There are two different types of hadrons 1] baryons such as protons or neutrons which are composed of three quarks and 2) mesons which are made of quarks and anti quarks. These quarks carry fractional electrical charges and they are spin half fermions. There exists different type of fundamental interaction between them called strong interaction. This interaction is possible because of the existence of a new degree of freedom i.e. color charge which was proposed by Greenberg in 1965. Strong interaction takes place by exchange of gluons. These gluons are spin one colored particles and are eight in number. The quantum field theory (Non Abelian gauge

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[*] Corresponding author: lax0123rad@gmail.com

theory) based on this interaction is known as Quantum Chromodynamics. This interaction between quarks are very much different from that of nucleons, when the distance between two nucleons is very small then the force between them is very large and as distance increases the force between them decrease and goes to zero, on the contrary but in case of two quarks, when the distance is very small then there is almost no force between quarks, this property is called asymptotic freedom, and when distance increase to 1 Fermi then the force becomes infinity and this region is called confinement region. The potential in QCD is $V(r) \sim \frac{-\alpha_s}{r} + \sigma r$ and potential QED is $(r) = \frac{-\alpha}{r}$.

Here α_s is the strong interaction coupling constant and sigma is called string tension. Thus, if we can push the quarks very much close to one another, then the confinement term of the potential is screened and all the bound states will melt down into quarks etc. This new phase of matter is called Quark Gluon Plasma. Quantum Chromodynamics is believed to be the theory which describes the strong interaction of quarks and gluons which are found in hadrons. The dynamics of both quarks and gluons are dictated by the QCD Lagrangian density

$$\mathcal{L} = \bar{\psi}\,(i\gamma_\mu D^\mu - M_{jk})\psi - \frac{1}{4} F^a{}_{\mu\nu}F_a{}^{\mu\nu}$$

Where the indices a refers to colors field $a = 1, 2, \ldots\ldots 8$ and $j, k, = 1, 2, 3$ and the covariant derivative acting on a quark field is defined as

$$D^\mu{}_{jk} = \delta_{jk}\,\partial^\mu + igT_{ajk}\,G_a{}^\mu$$

Here are the gluon field T_a are the color SU(3) generators g is the strong coupling constant and M_{jk} is the quark mass matrix. The gluon field tensor is

$$F^a{}_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - gf_{abc}G^b{}_\mu G_\nu{}^c$$

Where f_{abc} are the structure constants of SU(3) defined by the commutation relations of the SU(3) generators

$$[T_a, T_b] = if_{abc}T_c$$

The last term of (3) reveals the self-interacting nature of the gluon field. If the gauge groups are Abelain, then the structure constants f would all become zero and the gluon field would not be self-interacting. Thus the gluon self interaction have no analogy in the QED. In QED an electron under consideration attracts the particle of opposite charge and repels those of the same charge, it appeared by a cloud of charges of opposite sign in vacuum. Consequently the charge seen from a certain distance is less than the bare charge is as follows

$$\alpha(Q^2) = \alpha\{1 + \frac{\alpha}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right) + \left[\frac{\alpha}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)\right]^2 + \ldots\}$$

This leading log may be summed up and we get as

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}$$

As Q^2 increases, photon sees more of the bare charge and at some very large but finite Q^2

i.e the coupling constant becomes infinite.

In case of QCD description of the strong interaction of non Abelian gauge theory give where with N_c as the number of color is 3 and N_f as the number of flavor

of quarks. The other important consequence is that $\alpha_s(Q^2) \rightarrow \infty$ when $Q^2 \rightarrow \Lambda^2$ where Λ is
QCD scale factor ($\Lambda = 2000 MeV$). Thus r dependence of the coupling constant α_s is

$$\alpha_s(Q^2) = \frac{\alpha_s}{1 + \frac{\alpha_s b_0}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right)}$$

Therefore r dependence of the coupling constant α_s is $\alpha_s = \frac{1}{\frac{b_0}{2\pi} \ln\left(\frac{1}{\Lambda r}\right)}$
The coupling strength could be stronger as the distance r connecting q and $\bar{q}$ increased and the perturbation theory is not applicable. The exchange of gluon will pull towards each other and so the color lines of force are within the tube like region among the quarks. Therefore, these tubes have a constant energy density per unit length, the potential energy of the interaction increases with the variation $V(r) = \lambda r$ and the quarks and gluons can never escape from hadron. This is called infrared slavery and believed to be the foundation of confinement mechanism.
However, the discovery of Quark Gluon Plasma will give experimental proof for the asymptotic freedom and for the confinement mechanism in QCD. In the studies of QGP, we explore the answer of many vexing questions. What is the order of phase transitions? what are the values of critical temperature T_c and critical baryon chemical potentials μ_c at each energy ? what are the critical energy density and what are the signals of QGP formation. It has been assumed Universe was created after Big Bang and QGP ia at a very large energy and high temperature and density and we hope recreate situation like early Universe in the laboratory which was created for very short duration time (10^{-8}) sec after Big Bang. If we succeed to create QGP in the labortory, then we will be able understand the creation Universe. For this purpose a lot of research is going on. We are colliding two heavy ions in AGS at BNL, SPS at CERN, RHIC at BNL and LHC at CERN. In the studies of QGP we explore the answers of many vexing questions. What is order of the phase transition and what are the values of critical temperature T and critical baryon chemical potential at each energy.

1.1 Production of QGP in the Laboratory

It is known that in a high-energy nucleon-nucleon collision, the nucleons loose much of their energy and deposit the energy and deposit the energy in a small spatial region, thus creating a region of very large energy density. This results into creation of a large number of particles. A nucleon nucleon collision in nucleus nucleus collision is roughly additive in nature. Furthermore, because of the Lorentz contraction, the nucleon-nucleon collisions occur at about the same time and in about the same spatial region (in CM system). There is, therefore cooperative slowing down of the baryons and a nearly simultaneous production of overlapping domains of high energy densities to regions of very high energy densities. There are two different situation for the happening of such high energy density regions. In the stopping regime at about a few Gev per nucleon and in the CM system it occurs in the central rapidity region at a higher energy of about 100 GeV per nucleon. In such situations, we predict a collective slowing down of the baryons in the CM frame so that the nuclear matter is almost stopped in that frame and the type of QGP which may be formed in this region is a baryon rich QGP. At greater energies, the baryons cannot be absolutely stopped. They are slow down but still proceed forward in the CM frame after inelastic collisions. The baryons are well detached; the energy which is trapped between the colliding nucleon may become liberate in the region between retreating baryons the central rapidity region. The additive effect of numerous such colliding nucleons may generate a QGP with minute baryon constant.

1.2 Phase Transition of QGP

The deconfinement phase transition can occur in the laboratory by the following process
1) Compression of the nuclear matter i.e. puming more particle into the nuclear matter of constant volume. 2) Increasing the temperature of the system and creating
more pion bubbles. The first situation resembles the matter in the neutron stars formation and neutrons overlap on each other and finally a deconfined phase occurs.
Second situation is like a few micro seconds after the big bang. It is supposed that at high temperature, plasma of quarks and gluons is formed and it cool down to make a transition to a hadron gas phase. The phase transition at T= 0 is achieved by the compression of the nuclear matter and therefore, by increasing the net baryon density. We can get an order of magnitude of the compression factor needed for deconfinement the nuclear matter comparing the baryon density in nuclei and in nucleon. Volume of a nucleus of atomic number A is given $V = \frac{4}{3}\pi R^3$ with $R = 1.2A^{1/3} fm$ and the baryon density in the nucleon is $n = (\frac{4}{3}\pi r^3)^{-1}$ where r = radius of a nucleon by taking $r = 0.8 fm$, we could get $n = 0.45\ nucleon/fm^3$. However, we expect that critical baryon density is 1 $nucleon/fm^3$. So, in this simple geometrical picture, a compression factor of the order 3 is needed to bring the nucleon in the close contact. Compression factor of this order are expected to be achieved in ultra relativistic heavy ion collisions. In case of the QGP the number degree of freedom is considerably higher due to flavor, spin and color quantum numbers that are liberated. We have $n_Q = 2 \times 8 + 7/8 \times 2 \times 2 \times 2 \times 3 = 37$. In order to get an estimation of the crtical temperature we can use the bag model to describe the QGP phase transition. In this model we consider a bag of quarks and gluon with energy given by
$E = BV + C/R$ where B is bag constant, $\bar{V}$ is the volume of the bag and R is the radius of bag (momentum of quark inside the bag $\sim R$. At the minimum energy $\partial E/\partial R = 0$ which gives the radius as $R = \left(\frac{C}{4\pi B}\right)^{\frac{1}{4}}$. The mass of the bag is thus equivalent to the energy of the stable system is $M = E(R) = 16\pi BR^3/3$ or the energy density inside the bag is $4B$. In simple bag model, we find that the latent energy density to melt a bag is $4B$.

2. Entropy of QGP

Understanding about the thermodynamical behavior of quark gluon plasma is an important. Thermal partition function of canonical ensemble of quark gluon plasma is to be calculated by using the path integral formalism at the thermal equilibrium condition for the zero chemical potential. However, the quark gluon plasma is at temperature i.e $\beta = \frac{1}{K_B T}$

and here $K_B = 1$, $h = 1$ and

integrating over all quarks and gluon fields on $d = 4$ Minkowski spacetime is as follows

$$Z = \int D\overline{\Psi}\, D\Psi DG \exp\left(-\beta \int d^4x\, \mathcal{L}\right)$$

On the possibility of a phase transition between hadrons and quark gluon plasma.The partition function of an ideal quark gluon plasma system is to be calculated when the chemical potential is equivalent to be zero. We also calculated the entropy of canonical ensemble of quark gluon plasma in Minkowski spacetime with zero chemical potential is as follows

$$S = \log(\beta) - 1 + \log\left(\det(i\eth - m)\exp\left(\textstyle\sum_{n=1}^{\infty}\frac{1}{n}Tr\left(\frac{1}{i\eth - m}\right)(-igTG)^n\right) + \frac{4}{g^2} - \log\left(\frac{\beta g^2}{4}\right) - \log(\det(G \times G)).$$

However, the above entropy could able to explain thermodynamical behavior of the quark gluon plasma at finite temperature.

3 Conclusion

The field of Quark Gluon Plasma has grown at a surprisingly fast rate, attracting a large number of theorist and experimentalist from more sedate areas of nuclear and particle physics. The new field offers a very complex and rich domain for exploration. The developments are far from complete on the experimental investigations of properties and signals of QGP which started with availability of high energy ion beams in the late 1986. Theoretical efforts are also continued to devise unique and unambiguous signals to study the spacetime history of QG formed in such experiments as well as in the early Universe. The advent of LHC will vastly extend our range of accessible energy densities in nuclear collision. This will greatly help in reaching thermal behavior and deconfinement phase of hadrons. We can use statistical QCD in deriving the physical properties of such a thermodynamical systems produced in nuclear and astrophysics. In this paper we have calculated the entropy of Quark Gluon Plasma at finite temperature when the chemical potential is equivalent to be zero.

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